

Unification via Hyperfluid

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Abstract

Historic arguments against Aether theories disappear if the Aether is a 4D compressible hyperfluid in which each particle is our observation of a hypervortex formed in and comprised of hyperfluid. A particular Lagrangian for such a hyperfluid unifies gravity, QM, EM, particle behavior and spectra and more. The Lagrange equations of motion regenerate Maxwell's equations adding an equation for gravity, an equation for electric charge, and a coupling between gravity and EM forces to generate a discrete spectrum of hypervortex solutions that we observe as a spectrum of particles. In the hyperfluid theory, quantum phenomena result from detailed hypervortex behavior as observed in our 3D universe. Waves within and along the hypervortexes hold quantum information and resolve the contradiction between relativity and "spooky action at a distance". Gravity results from gradients in the fluid density near vortices. Light is transverse waves in free space whose behavior explains why we observe a 3D universe as a curved slice through the 4D hyperverses. Clock rates depend on fluid density, vortex motion, and the mechanism by which we observe only three of the four spatial dimensions, thus intertwining gravity, clock rates and quantum phenomena.

Keywords: gravity, relativity, Aether, hypervortex, unification, EPR paradox, hyperfluid,

Introduction. The EPR paradox,ⁱ presented in 1935 by Einstein, Podolsky and Rosen, introduced a fundamental contradiction between General Relativity and Quantum Theory. General Relativity considered the speed of light as a limit so fundamental as to be tied to curvature of space itself. In contrast, Quantum Theory required faster than light information exchange. Over the decades, the Quantum Theory prediction has been shown to be correctⁱⁱ (or likely so depending on one's interpretation of the results to date); however, our understanding of gravity has yet to be modified accordingly. This work makes fundamental changes to address the paradox and thus improve unification.

Since quantum mechanical faster-light-information exchange appears to be confirmed, hence this work reconsiders whether light's behavior is tied to anything as fundamental as curvature of space. The alternative is that the behavior of light is explained by the main alternative hypothesis -- propagation in a medium. Of course, that alternative was dismissed early in the 20th century due to results of the Michelson-Morley experiments. Thus, to address the EPR paradox, we review that history.

The Michelson-Morley experimentsⁱⁱⁱ were predicated on the existence of a boundary between particles and Aether, the then believed medium for light propagation and since Medieval and even Ancient times depending on one's reading of history. Their experiment was devised to detect and measure boundary effects; thus their negative results indicate a lack of any fluid-particle boundary.

Main stream analysis^{iv} has claimed that the Michelson-Morley results imply the absence of Aether; however, Einstein's principle of "Wave-Particle Duality" provides foundation for another possibility. In particular, it suggests that both light and particles are comprised of fluid. If so, there would be no boundary discontinuity, which explains the negative results of the Michelson-Morley experiments. Such a fluid would not be Aether, as Aether was specifically a fluid that filled the space between particles. Fluids in general support two types of structures – waves and vortexes. Since light has long been considered to be a wave in almost any theory, this work explores particles as vortexes of fluid.

The above analysis suggests a Lagrangian to describe fluid that meets the basic criteria for unification. The Lagrangian is constructed in flat Euclidean coordinates, since the above analysis voids the need to curve space. It uses four spatial dimensions, since no solution using only three spatial dimensions has been found. It uses global time realizing that in this model the observed clock rate variations may not be fundamental but rather effects due to clock construction from particles which, being made of fluid, may have rates that depend on fluid's the local properties. Similarly, rulers, being made of particles which are made of fluid, may vary over space. Ergo, Special Relativity and General Relativity will be observables, artifacts of our methods of observation.

Hyperfluid Lagrangian. The Lagrangian in Equation (1) is constructed as the integral of a Lagrangian density over the four spatial dimensions. Thus, the fluid is a hyperfluid. The hyperspace is considered real. Our observable three-dimensional universe is a moving, changing curved slice through hyperspace. The hyperspace is filled with observable three-dimensional universes, mutually unobservable due to our means of observation, discussed later. Also, the Lagrangian is covariant and supports the principle of relativity (e.g. that one cannot determine ones motion from local measurements.) However, it includes no explicit terms for observed relativistic behaviors; those arise from our observation mechanisms.

$$L = \int \left(\frac{P_\mu^2}{2M} - \frac{k_V}{2} \left(\frac{\partial P_\mu}{\partial x_\nu} - \frac{\partial P_\nu}{\partial x_\mu} \right)^2 - \frac{k_G}{2M} \left(\frac{\partial M}{\partial x_\mu} \right)^2 + \lambda \left(\frac{\partial M}{\partial t} + \frac{\partial P_\mu}{\partial x_\mu} \right) \right) d^4 x_\mu \quad (1)$$

In the Lagrangian, the symbol M represents a scalar field denoting the density of the hyperfluid at each location in space. The symbol P_μ represents a vector field denoting the momentum of the hyperfluid at each location in space. The symbol λ represents a scalar field, a Lagrange multiplier. The symbols k_V and k_G are constants whose quantified values will be determined by matching to data.

The first term inside the integral represents the kinetic energy of the hyperfluid at each location in space. The second term represents the corresponding potential energy due to transverse gradients of the hyperfluid momentum. The third term represents the additional potential energy due to gradients in the hyperfluid density. The fourth term inside the integral is the typical holonomic constraint used to enforce continuity. Continuity, as used here, requires that in order for hyperfluid to get from one location to another it must move there; it cannot just disappear from one location to appear in another.

Standard Lagrange methods are used to derive the equations of motion from Equation (1). This generates a vector equation (Equation (2)) and a scalar equation (Equation (3)). In both, the terms that include λ and also terms without derivatives have been moved to the right side of the equation.

$$2k_V \frac{\partial}{\partial x_v} \left(\frac{\partial P_\mu}{\partial x_v} - \frac{\partial P_v}{\partial x_\mu} \right) = \frac{\partial \lambda}{\partial x_\mu} - \frac{P_\mu}{M} \quad (2)$$

$$\frac{k_G}{M} \frac{\partial^2 M}{\partial x_\mu^2} - \frac{k_G}{M^2} \left(\frac{\partial M}{\partial x_\mu} \right)^2 = \frac{\partial \lambda}{\partial t} + \frac{P_\mu^2}{2M^2} \quad (3)$$

Maxwell's Equations in the Hyperfluid. Equation (2) is equivalent to Maxwell's equations. In particular, aside from a choice of units (e.g. MKSA or Gaussian), appropriate definitions transform the vector equation into the covariant form of Maxwell's equations. First, mapping the electromagnetic vector potential as $A_\mu \equiv 2k_V P_\mu$ makes the vector potential proportional to the hyperfluid's momentum and completes the transform for the left side of Equation (2).

Transformation of the right side of Equation (2) is achieved by equating it to a charge current $J_\mu \equiv \rho V_\mu$. Here ρ is the electric charge, and V_μ is the physical velocity of the hyperfluid. Each is adjusted for the presence of four spatial dimensions to complete the mapping of Equation (2) to Maxwell's Equations. Regarding ρ , to satisfy Gauss's Law with four spatial dimensions, point charges must be replaced with lines of charge. Thus, in the hyperfluid model, vortices have substantial length and, since that length is unobserved, it must be along the unobserved spatial coordinate, the w coordinate in Euclidean (w, x, y, z) hyperspace. Charge on a hypervortex becomes a quantity per unit length, and the charge on an observed particle is the charge on the length of hypervortex that lies within one observable universe. Regarding V_μ , $V_\mu = P_\mu/M$ as in classical physics; no relativistic correction is required or used. The V_w component is the net velocity along w of the hypervortex relative to the observable universe. These understandings of ρ and V_μ enable the definition of charge per Equation (4), and which transforms Equation (2) into Equation (5) which is almost exactly a covariant form for Maxwell's equations.

$$\rho \equiv \frac{V_w}{4\pi} \left(\mathbf{1} - V_\mu^{-1} \frac{\partial \lambda}{\partial x_\mu} \right) \quad (4)$$

$$\frac{\partial}{\partial x_v} \left(\frac{\partial A_\mu}{\partial x_v} - \frac{\partial A_v}{\partial x_\mu} \right) = \frac{4\pi}{V_w} J_\mu \quad (5)$$

Equation (4), besides transforming Equation (2) into Maxwell's equations, provides a means to compute the charge for any hypervortex from the other structural properties of that hypervortex. Thus, in the hyperfluid model, charge need not be added to the theory as an independent property of the universe.

Regarding Equation (5), the residual difference between it and the standard covariant form of Maxwell's equations^v disappears if we make the substitution $V_w = c$, where c is the speed of light. That substitution is not made here in part to highlight the source of that term in the equation and in part because, in the hyperfluid model, the speed of light need not be a constant.

Gravity in the Hyperfluid. Equation (3) is a new equation, an equation for gravity. To see this, consider homogeneous solutions to the scalar equation, i.e., solutions to a variant of Equation (3) in which the right side is set to zero. Ignoring the trivial solution in which M is a constant, a more interesting solution is given by Equation (6). Its validity can be checked by substitution. In Equation (6), α is any constant

value, M_∞ is the density of the hyperfluid away from the gravitational well, and r is the three-dimensional distance ($r^2 \equiv x^2 + y^2 + z^2$) from the origin of the gravitational well. M in Equation (6) is independent of the w coordinate, in accord with the long length of each particle's hypervortex along the w coordinate.

$$M(r) = M_\infty \exp\left(-\frac{\alpha}{r}\right) \quad (6)$$

Equation (6) defines an exponential gravitational well whose size scales with α and which addresses multiple residual concerns that exist in other theories of gravity. Also, it does so while retaining the same behavior as the $1/r$ form for gravity for large values of r because at large values of r , the dominant non-constant term in a polynomial expansion of the exponential form is the $1/r$ term.

The form for gravity that is given in Equation (6) addresses an issue that occurs at $r = 0$ in other theories. In particular, whereas the standard $1/r$ form for gravity is discontinuous at $r = 0$ and thus violates the core principles of relativity; the hyperfluid's exponential form in Equation (6) is continuous and continuously differentiable even at $r = 0$ and satisfies the principle of relativity.

Equation (6) addresses gravity theory issues regarding the relation between gravitational mass and inertial mass. Whereas for wells of the $1/r$ form, the gravitational energy computes as infinite in complete disagreement with reality, the energy of exponential gravitational wells scales as α , in accord known reality. To compute gravitational energy in the hyperfluid model the potential energy term $\frac{k_G}{2M} \left(\frac{\partial M}{\partial x_\mu}\right)^2$ from the Lagrangian in Equation (1) is integrated over r using $M(r)$ from Equation (6).

The standard mathematical relation $\frac{\partial \ln(Y(x))}{\partial x} = \frac{\partial(Y(x))}{\partial x} / Y$ is used to transform the potential energy term into $\frac{k_G M}{2} \left(\frac{\partial \ln(M)}{\partial x_\mu}\right)^2 = \frac{k_G}{2} \exp\left(-\frac{\alpha}{r}\right) \left(\frac{\alpha}{r^2}\right)^2$. Integrating this over r from zero to infinity gives the energy of the gravitational well as $\left[\frac{k_G}{2} \alpha \exp\left(-\frac{\alpha}{r}\right)\right]_0^\infty = \frac{k_G}{2} \alpha$. Ergo, the well's energy scales with its size, α .

The benefits of the hyperfluid model approach to gravity extend to issues regarding forces created by multiple gravitational wells. The gravitational potential due to multiple masses is experimentally found to be additive and that result is included as an assumption in typical gravitational theories. The hyperfluid model removes the need for such ad hoc assumption. Rather it becomes derivable from Equation (3). The derivation starts with Equation (7) which provides an exact homogeneous solution to Equation (3) for any number, n , of gravitational wells. In Equation (7), α_i and r_i represent the size and location, respectively of the i^{th} gravitational well.

$$M(r) = M_\infty \prod_{i=1}^n \exp\left(-\frac{\alpha_i}{|r-r_i|}\right) \quad (7)$$

Equation 7 shows that, in general, gravitational potentials of multiple masses are multiplicative. Still, at locations sufficiently far from all of the masses (i.e. $|r - r_i| \gg \alpha_i$, for all i) the gravitational potential becomes additive. This can be derived by polynomial expansion of Equation (7), as in Equation (8).

$$M(r) \approx M_\infty \prod_{i=1}^n \left(1 - \frac{\alpha_i}{|r-r_i|}\right) = M_\infty \left(1 - \sum_{i=1}^n \frac{\alpha_i}{|r-r_i|}\right) \quad (8)$$

Particles and Hypervortexes. Quantification of specific particle properties in the hyperfluid model awaits solutions to the full inhomogeneous coupled Equations (2) and (3). Such solutions will be challenging; the existence and complexity of hypervortex solutions is suggested from study of other compressible fluids. Air provides particularly useful insights as the large physical size of vortices within air provides opportunity to see some of their details. Also, major vortexes in air, including tornadoes and hurricanes have been much studied computationally. To solve for particle properties in the hyperfluid model, it may be helpful to analogize leptons to tornadoes and baryons to hurricanes, though the analogy will not be exact due, for example, to the hyperfluid's extra dimension and different Lagrangian. The mathematics of string theory or M-theory may provide methods, symmetry rules and other insights for solving the equations. To obtain solutions that match known particle properties may require addition of terms to the Lagrangian that are significant only where $M \ll M_\infty$. For example, where the hyperfluid density approaches zero, there may be surface tension.

A hypervortex solution comprises determination of the fields M , P and λ which can then be used to compute particle properties. Equation (4) provides the relation for computing charge. Also, the particle mass will correspond to the combined kinetic and potential energies of the hyperfluid that lies within the hypervortex. Strong forces and weak forces will also derive from the hypervortex solutions. Derivation of particle properties from hypervortex properties will leverage the idea that a particle is the portion of a hypervortex with lies within one observable universe. Further, we expect that values for k_G and k_V can be determined by comparison between data and computed results.

Special and General Relativity in the Hyperfluid. Figure 1 is used to discuss the observation of General and Special Relativity and, later, the EPR paradox and Quantum Mechanics in the hyperfluid model. The figure shows a variation of a Feynman Diagram. The principle alteration is that whereas in a Feynman Diagram one axis represents time, in Figure 1 that axis represents the w coordinate. This distinction is important to the proper understanding of the figure.

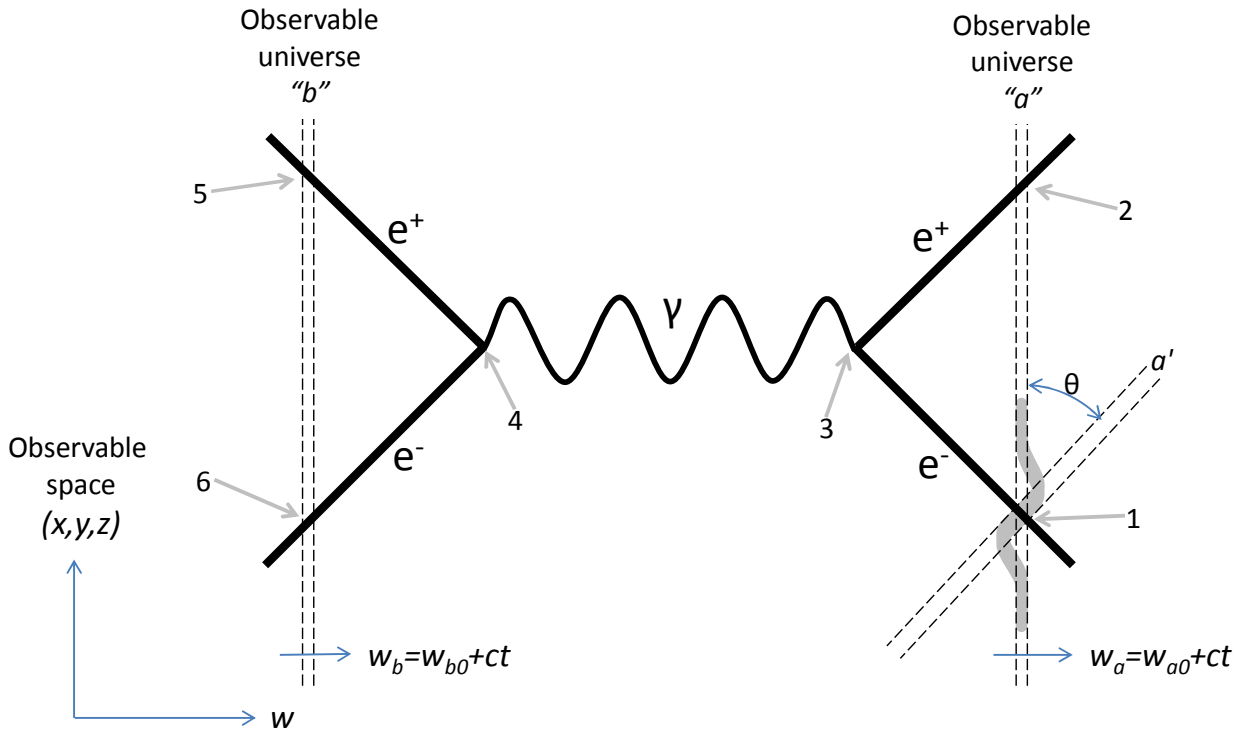


Figure 1. Hyperfluid Variant of a Feynman Diagram.

The figure can be thought of as a snapshot in time of hypervortices colliding to annihilate at Location "4" in the figure, there creating a photon γ in the hyperfluid which travels some distance and which then creates a new pair of hypervortices at Location "3". The hypervortices move over time. Also, they may come into existence and go out of existence over time. The photon γ also has such behaviors.

Added to the diagram are observables. Two observable universes are shown, labeled "a" and "b". They are shown at the time of the snapshot that is the figure. They move to the right in the figure over time at the local speed of light. They do so because the hyperfluid flows to the right at that speed. Observers in *a* will have seen events evolve over time something like those shown in the diagram. Observers in *b* may or may not observe similar events depending on whether the e^+e^- particle annihilation at location "4" is still in existence as *b* continues to move to the right over time. Each observable universe is represented by two dashed lines to represent a thickness of the observable universe along the *w* coordinate. Between observable universes *a* and *b* are a continuum of additional observable universes that are not shown; every location in the hyperfluid is in an observable universe.

The motion and behavior of the hypervortices in the figure complements the motion of the observable universes such that General and Special Relativity are observables. The details shown at the location labelled "1" in the figure are used to show this. Shown in gray is a deformation of the observable universe caused by the interaction of hyperfluid within the hypervortex with the laminar hyperfluid flow outside the vortex. Similar deformations occur at Locations "2", "5", and "6" in the figure, but are not shown. At the intersection of the hypervortex and the observable universe is the observed object that we call a "particle".

Special Relativity and General Relativity derive, or try to derive, the observed effects of the deformation shown at Location “1” without addressing the physical existence of the hyperfluid, the w coordinate, the hypervortexes or the effect that the deformation has on clocks and rulers. The hyperfluid model’s recognition of all of these effects explains the observed effects and their causes, all while also enabling unification. As shown in Figure 1, the deformation keeps the observable universe locally perpendicular to the hypervortex for all angles of the hypervortex relative to the observable universe a . Thus, the reference frame at the hypervortex is locally indistinguishable from other frames, in accord with the principle of relativity. An observer inside the deformation might assume that the observable universe extends along a' in the figure, but that would be wrong. That observer’s frame of reference has limited size and any calculations made using that reference frame apply only within that limited domain.

Special Relativity provides relativistic mass as $m = \gamma m_0$ where $\gamma \equiv (1 - v^2/c^2)^{-1/2}$ and m_0 is the particle’s rest mass. In the Lorentz transform (Lorentz), $\gamma = 1/\cos(\theta)$, where θ is a mathematically defined rotation angle without physical meaning. The hyperfluid model adds physical meaning to that θ . Specifically, it shows that θ is the angle between a and a' in Figure 1. Further, in the hyperfluid, by simple geometry in Figure 1, it is clear that the factor $1/\cos(\theta)$ corresponds to the relative length of hypervortex that lies within the observable universe as a function θ . Thus, in the hyperfluid model, $m = \gamma m_0 = m_0/\cos(\theta)$ applies because physically the length of hypervortex in the observable universe varies as $1/\cos(\theta)$.

The Lorentz transform also provides $\beta \equiv v/c = \sin(\theta)$. In Figure 1, if the hypervortex at location 1 does not move over time as the observable universe moves, then the observed velocity, v , will NOT satisfy $\beta = \sin(\theta)$. Motion of the hypervortexes along the w coordinate provides physicality that satisfies the complete Lorentz transform.

General Relativity tries to represent the deformation in the observable universe near a hypervortex with curved space-time coordinates without including a w coordinate. In particular, it includes: (1) the combined effects due to changes in hyperfluid motion at the deformation, (2) variations in hyperfluid density at the associated gravity well, and (3) the effects of the deformation on local clocks and rulers. The hyperfluid model, by adding hyperfluid as a physical foundation and adding the w coordinate to enable describing the deformation as a curved surface in a flat hyperspace, exposes the physicality underlying the mathematics of General Relativity.

EPR Paradox. Referring to Figure 1, Relativity theory says that information sent from “1” to “2” travels at the speed of light. Quantum Mechanical tests show, or appear to show, that certain information links “1” and “2” instantaneously, or nearly so. In Figure 1, the portion that includes the labelled Locations “1”, “2”, and “3” is sufficient to explain how and when each is true.

First, consider the flow of information from Location “1” to “2” via electromagnetic (EM) waves. The hypervortex at “1” emits EM waves in the direction perpendicular to the hypervortex. Thus the EM is emitted into the deformation in the direction towards a' . Its path curves to follow the deformation into observable universe a and arrives at location “2”. The vast majority of its travels are in free space where only transverse electromagnetic waves exist, and which Maxwell’s equations properly indicate travel at

the speed of light. Thus, the EM waves travel from “1” to “2” at the regional speed of light and always within the observable universe. This behavior of light is the reason that the hyperverses are observed in mutually unobservable three-dimensional observable universes. It is also this effect that causes observation of the speed of light as a speed limit.

Now consider a Quantum Mechanical type action. At “1” a measurement of the particle puts a strain on the hypervortex. That strain propagates from “1” to “3” as a wave along the hypervortex and then similarly continues from “3” to “2”. The propagation speed of that wave is not constrained by Maxwell’s Equations.

Computation of the speed of waves propagating along a hypervortex requires computing hypervortex solutions to the Lagrangian, yet there is already good indication that that speed will be much greater than the speed of light. In particular, it is shown above that particle mass and particle energy correspond with hypervortex length, and that knowledge combines with the principle of energy conservation and with measurements that show that all electrons have the same mass, to infer that the hypervortex is very stiff as regards changing its length. That stiffness translates into very high speed of longitudinal waves traveling along the length of a hypervortex.

A real world example shows the potential speed of longitudinal waves along hypervortexes compared to transverse EM waves. In water, transverse waves, which we can visually observe, occur at the surface boundary of the water and travel at speeds ranging from a few kilometers per hour to hundreds of kilometers per hour^{vi}. In contrast, the longitudinal waves that we cannot visually observe propagate under the water’s surface at about 5,000 kilometers per hour^{vii}. A similar speed ratio between transverse EM waves and longitudinal waves within hypervortexes is reasonable to expect.

Quantum Mechanics. Two key aspects of quantum mechanics are its probabilistic nature and faster-than-light information flow among “entangled” particles. Regarding entanglement, the hyperfluid model provides physicality for the concept of “entanglement”. As described above in the explanation of the EPR paradox, faster-than-light information flow occurs via propagation along hypervortexes. Thus, in Figure 1, two particles are entangled if their hypervortexes are connected as in Location “3”. Also, the figure suggests that entanglement ends if and when the connection breaks between the e^+ and e^- hypervortexes at Location “3”.

Regarding the probabilistic nature of Quantum Mechanics, the hyperfluid also provides physical mechanism for that. First, the calculation of the temporal evolution of a system requires knowledge of the current state of that system. However, no matter how precisely we know the current state of our observable universe, we have no current method to measure the precise state of other observable universes whose states and behaviors impact that calculation. Second, and related, the EPR discussion indicates the presence of waves traveling along the hypervortexes. These waves add uncertainty to the location and position of the observed particles. These two factors (and perhaps other factors) limit the accuracy of our calculations, and Quantum Mechanics is in part a method to adapt to that limitation.

The relation between quantum mechanical commutation properties and our uncertainty regarding the current state outside of our observable universe follows from concepts of the 1960’s.^{viii} Both Feynman

and Schwinger developed arguments and mathematics showing that quantum mechanical commutation properties arise naturally in Lagrangian mathematics, if and when the Lagrangian has need to be evaluated over variant paths. The hyperfluid model provides physical reason for evaluating over variant paths. In particular, in the hyperfluid model those variant paths are the possible physical paths of hypervortexes along w . In particular that includes all paths are consistent with the observed behavior within our observable universe. Because of our lack of information along w , we compute the temporal evolution of that system from the current state to a future state as a transition probability. Computation of that probability distribution requires consideration of alternate hypervortex paths along w . The calculation is thus subject to Feynman's and Schwinger arguments. In summary, quantum mechanical commutation properties result from our lack of knowledge regarding hypervortex initial conditions beyond our observable universe.

Dark forces. Introduction of the term "dark matter" reflects the discovery that galaxies hold their shape despite the lack of sufficient known matter within them to provide gravitational forces to support that cohesion^x. The hyperfluid model provides physical mechanism for that cohesion, and it does so using only the aspects already introduced in the model. In particular, galaxies look like enormous vortexes and the hyperfluid model supports the possibility of such vortexes. There is no known upper limit for vortex sizes for fluids described by the Lagrangian presented in Equation (1). Thus, in the hyperfluid model galaxies can exist regardless of any insufficiency of mass of smaller hypervortexes within the galaxies.

Introduction of the term "dark energy" reflects the discovery that our observable universe is expanding at an accelerating rate for no known reason^x. The hyperfluid model provides a physical reason without need to modify the model. Per the hyperfluid Lagrangian, if our observable universe is heading into a region of reduced hyperfluid density, that gradient in the hyperfluid density along w will generate a force that accelerates the hyperfluid flow along w . We observe that effect as an accelerating expansion. Similarly, if our observable universe were heading into a region of increased hyperfluid density, the universe expansion rate would be observed as decelerating, while if the hyperfluid density were uniform along w , then the expansion rate would be observed as constant. Thus, our observation of an accelerating expansion rate suggests that the hyperfluid density is decreasing along w .

Conclusions. The hyperfluid model takes us from a particle centric path towards unification that has been the mainstream for one century and returns us to a fluid centric path that had been the mainstream for much longer. In doing so, it provides a fundamental resolution to the EPR paradox, plus a Lagrangian and equations of motion in accord with the basic goals for unification. Further, it provides physical mechanisms to explain various previously abstract concepts each in need of the clarity that such physical mechanisms provide. The model reduces the number of independent axiomatic assumptions required. For example, Equation (4) replaces the need to add electric charge as a separate concept in the model, while Equation (8) replaces the need to assume that gravity is additive.

The work presented here may be viewed as a framework model for completing unification. Additional derivations from the hyperfluid model have been done that are yet to be published in journals. At the same time, much additional work remains. For example, efforts to compute specific hypervortex solutions to the Lagrangian and to explore the possibility of addition of terms in the Lagrangian have

barely begun. Also, independent calculation of the model's results for the advance of the perihelion of Mercury and other specific test cases is desirable. Finally, a measurement of the speed of information propagation between entangled particles would provide important evidence supporting the approach.

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